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Bundling, Vertical Differentiation, and Platform Competition

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Abstract: This paper studies the bundling strategies of two firms that each sell a horizontally differentiated platform and a complementary good. When the complementary goods are vertically differentiated, the firm that sells the superior one can commit to a more aggressive pricing strategy through bundling. In the presence of asymmetry in externalities between the two sides in the platform market, bundling may be profitable without foreclosing the rival when platforms implement cross subsidies from the high-externality side (developers) to the low-externality side (consumers). Bundling has a positive effect on welfare because it allows for better internalization of the indirect network effects and reduces the developer cost of multi-homing, but it also has a negative effect because some consumers consume less-preferred components. Consequently, bundling is socially desirable when platforms are not too differentiated and the vertical differentiation between the complementary goods is high.

Keywords: bundling; two-sided platforms; vertical differentiation.

JEL codes: L1; L42; D4; D84.

1 Introduction

In the market for mobile application stores, major competitors such as Google and Apple bundle application stores with their in-house operating systems (OSs). Samsung takes a different approach: Its Galaxy Apps store runs on Google's Android, instead of Tizen, its in-house OS.¹ It is worth noting that Samsung chooses not to bundle the Galaxy Apps store with Tizen to form a proprietary

¹ See <http://www.businessinsider.com/samsung-unleashes-tizen-store-to-the-world-2015-5>, accessed October 2016.

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ecosystem, even though the Galaxy Apps store is struggling to gain market share against Google Play.² There is clearly a vertical difference between Tizen and Android.³

Bundling a platform with a complementary good is common in industries where two-sided market structures and quality differences are prevalent. However, how the quality advantage is leveraged from a market to a multi-sided market has seen little investigation. This paper helps to address this issue by studying two-product firms' pure bundling strategies and the consequences on competition and welfare as firms engage in duopolistic competition in a two-sided platform market (e.g. Google Play vs. Galaxy Apps store) and a complementary good market (e.g. Android vs. Tizen). I highlight horizontal differentiation in the former market and vertical differentiation in the latter.

As noted in Whinston (1990), bundling works as a commitment device. I show that the firm that sells the superior complementary good can effectively commit to a more aggressive pricing strategy through bundling. Bundling allows the firm to commit to a lower price for the bundle on the consumer side, consequently expanding demand and increasing profit on the developer side. In the presence of asymmetry in externalities between the two sides, bundling may be profitable for the bundling firm without foreclosing the rival when platforms implement cross subsidies from the high-externality side (developers) to the low-externality side (consumers). When the vertical differentiation between the complementary goods is high, bundling leads to the foreclosure of the rival firm.

The results of this paper provide an explanation of observations from the mobile application store market: why Google and Samsung take such different approaches to bundling, and why Galaxy Apps store has such a small market share. The results also explain why a firm (e.g. Samsung) that owns an inferior complementary good (e.g. Tizen) may not want to build a proprietary ecosystem by bundling the platform with the complementary good. Bundling gives the firm that sells the superior complementary good a competitive advantage when competing with a horizontally differentiated platform, and this advantage cannot be eroded by the bundling practice of the firm that sells the inferior complementary good.

² Galaxy Apps store had 24 million monthly active users in April 2017. See <https://www.mobilemarketer.com/news/samsungs-app-store-seeks-to-double-audience/441287/>, accessed June 2018. Google Play had 1 billion monthly active users in October 2017. See <https://venturebeat.com/2017/10/19/google-play-hits-8-billion-new-app-installs-per-month-and-gets-new-developer-features/>, accessed June 2018.

³ See <https://appleinsider.com/articles/17/04/05/researcher-calls-samsungs-tizen-os-the-worst-code-ive-ever-seen>, accessed June 2018.

I conclude that bundling need not harm social welfare. This paper contributes to the recent discussion on Google's potential anti-competitive behavior and related antitrust cases. Google effectively implements bundling of its apps, including Google Play, through Android's Mobile Application Distribution Agreements contracts with manufacturers (Edelman 2015). Google also deploys a non-compete clause in its Google Play Developer Distribution Agreement to block third-party application stores from appearing in the Play Store,⁴ creating more obstacles for those who wish to install third-party stores.⁵ In this paper, bundling by a superior complementary good improves social welfare in two ways. First, it allows the bundling platform to better internalize the indirect network effects. Second, it decreases the mass of multi-homing developers, and hence the cost of multi-homing. However, bundling also leads to three sources of inefficiency. First, the alternative system generates a utility loss due to lower participation on both sides of the platform. Second, some consumers consume the less-preferred platform. Third, the alternative system consists of the inferior complementary good. Thus, bundling improves social welfare if platforms are not too differentiated and there is high differentiation between the complementary goods. However, it is important to note that the benefits of bundling discussed in this paper largely stem from the indirect network effects as bundling raises participation on both sides of the bundling platform. Bundling can generate a tipping toward the bundling platform, and the positive feedback loop between the two sides amplifies the tendency to tip. Consequently, bundling limits consumers' choices. Thus, I suggest that competition authorities should scrutinize bundling practices in detail when two-sided markets are involved.

⁴ See <https://play.google.com/about/developer-distribution-agreement.html> Section 4.5, accessed July 2016.

⁵ In June 2014, Aptoide filed a complaint with the European Commission (EC) accusing Google of abusing its position in the smartphone market to push users away from independent application stores (see <http://fortune.com/2014/06/17/google-faces-new-antitrust-complaint-in-europe-from-portuguese-app-store/>, accessed September 2016). Aptoide claimed that Google had created obstacles for users to install third-party application stores onto Android and bundled services that were essential to Android with Google Play (see <http://www.wsj.com/articles/google-faces-fresh-antitrust-complaint-in-europe-1402941192>, accessed October 2016). Consumers need to download the Android application package to install a third-party application store, then enable the installation from "Unknown Sources" in Android settings. Even if consumers are willing to go through this hassle, they are often unable to get rid of Google Play, because without it, they would experience difficulty getting certain apps from Google or others. "Apps from Microsoft and Google will be available in the Play Store, while Samsung's apps will always be available for download through its Galaxy Apps marketplace" (see <https://www.sammobile.com/2015/03/23/some-pre-installed-apps-on-the-galaxy-s6-and-the-galaxy-s6-edge-can-be-deleted/>, accessed July 2017).

Although tying would not emerge as an equilibrium strategy in the setting absent the non-negative price constraint,⁶ a mandated tying scheme with a price cap for the stand-alone sale of the superior complementary good can improve social welfare. The price cap requires that the firm not raise the price of the superior complementary good under tying above the equilibrium price under separate selling. Regarding the anti-competitive bundling behavior of Google, offering competing apps (including app stores and search engines) equal access to pre-installation on Android improves social welfare.

The rest of the paper is organized as follows. In Section 2, I review the literature that is closely related to the present study. Section 3 presents the setup of the model. I study firms' bundling strategies and the impact of bundling on welfare in Section 4. In Section 5, I show that the insights obtained in the main model are robust. Section 6 concludes.

2 Related Literature

In Whinston (1990), a two-product firm ties its monopoly product to its product in a duopoly market and becomes more aggressive in the duopoly market. Tying works as a commitment device. He shows that pure bundling reduces equilibrium profits of all firms; hence, it is usually adopted to deter entry into the differentiated market. However, the logic may change when the differentiated market is two sided. First, platforms may use cross subsidies to coordinate two-sided demands (Rochet and Tirole 2003; Armstrong 2006). Tying or bundling can be used to subsidize one side and may be profitable. Second, when there is additional profit from the other side at stake, the rival may behave differently in response.

This paper contributes to the literature that studies tying and bundling in two-sided markets. Inspired by the tying of Windows Media Player to Windows, Choi (2010) studies the effects of tying on market competition and social welfare, emphasizing the role of multi-homing on both sides of the market. Tying can be welfare-enhancing because multi-homing reduces the exclusionary effect of tying. de Cornière and Taylor (2017) study a setting in which a multi-app developer competes with a single-app rival for inclusion on devices. Bundling of

⁶ The difference between pure bundling and tying is that the tied good is still available on a stand-alone basis (Tirole 2005). Accordingly, consumers who do not consume the bundle would still have access to the superior complementary good under tying in this paper.

apps can reduce differentiation between device manufacturers and hence benefit consumers.⁷

The present paper is closely related to Amelio and Jullien (2012) and Choi and Jeon (2016). In Amelio and Jullien (2012), platforms want to set negative prices on one side and use tying to implement implicit subsidies because of the non-negative price constraint. Consequently, tying increases profits and consumer welfare for a monopoly platform by better coordinating two-sided demands. In a duopoly setting, tying has a strategic effect on competition: Tying on one side makes a platform more or less competitive on the other side, depending on the externalities of each side. There are a few key features differentiating this paper from Amelio and Jullien (2012). First, their paper compares the effects of tying between monopolistic and duopolistic settings, whereas this paper focuses on leveraging quality advantage from one market to a two-sided market. Second, in their setting pure bundling only arises if a consumer's valuation of the bundled good is below the marginal cost. In this paper, consumers' willingness to pay for the bundled product needs to be positive for bundling to be effective. Third, prices should be interpreted as markups and can be negative in this paper.

Choi and Jeon (2016) emphasize the interplay between tying and the non-negative price constraint. A two-product firm can tie to leverage its monopoly power from one market to a duopoly platform market as it competes against a superior platform: Tying allows the firm to relax the non-negative price constraint on the consumer side and capture the profit from the other side. The non-negative price constraint limits the rival platform's response, thus working in favor of profitable tying. Three key features differentiate this paper from Choi and Jeon (2016). First, prices can be negative in this paper, so the rival firm's response to bundling is not limited. Second, this paper emphasizes the quality advantage of the bundled good. The profitable bundling result is induced by leveraging the quality advantage to the platform market. Third, in my paper the platforms are horizontally differentiated to consumers, highlighting the business-stealing effect on the single-homing side and the market-expansion effect on the multi-homing side.

The model of competition between systems I use is similar to that of a few papers that investigate competitive bundling within a symmetric market structure. In Matutes and Regibeau (1988, 1992), two-component firms make compatibility decisions before engaging in duopolistic price competition absent network

⁷ Bundling in de Cornière and Taylor (2017) is imposed by app developers on device manufacturers but not consumers.

effects.⁸ Gans and King (2006) study interfirm bundling of two independent products, each produced by two horizontally differentiated firms. They show that two firms, each offering a different product, have an incentive to offer consumers a bundled discount. Bundling converts independent products into complements. Brito and Vasconcelos (2015) modify the model of interfirm bundling with vertical differentiation and show that pairs of firms producing goods of the same quality offer joint discounts in equilibrium. Avenali et al. (2013) study how the bundling firm uses mixed bundling to affect its competitors product quality.⁹ None of these papers consider two-sidedness, while this paper highlights vertical differentiation in the complementary good market and horizontal differentiation in the two-sided platform market.

3 The Model

Consider two markets: the application store market (market A) and the OS market (market B). Two firms, labeled 1 (e.g. Google) and 2 (e.g. Samsung), each have a product in each market.

In market A , there are two platforms that facilitate the interactions between consumers and developers. A_1 (e.g. Google Play) and A_2 (e.g. Samsung Galaxy Apps) are exogenously located at the two ends of a Hotelling segment, respectively—i.e. $x = 0$ and $x = 1$. Let $p_{A_i}^C$ and $p_{A_i}^D$ denote the prices platform A_i charges consumers and developers, respectively, where $i \in \{1, 2\}$. Let $n_{A_i}^C$ and $n_{A_i}^D$ denote the masses of consumers and developers on A_i , respectively. In market B , there are two vertically differentiated complementary goods. Let q_{B_i} and p_{B_i} denote the quality and price of B_i , respectively. B_1 (e.g. Android) has quality z , and B_2 (e.g. Tizen) has quality $z - \delta$, where $0 < \delta \leq z$. The marginal costs of the platforms and the complementary goods are normalized to zero. Prices in this paper should be interpreted as markups, and therefore they can be negative. Firms can decide whether to sell the two components separately or as a bundle, and there is no cost advantage to bundling. Assume that consumers can assemble a system that is close to their ideal if no firms introduce bundling. If a firm practices pure bundling, it is not possible for consumers to buy its bundle then drop one of the components and use the other one instead. Thus, bundling can be interpreted as a compatibility decision.

⁸ In Matutes and Regibeau (1988), firms must price the whole system equal to the sum of their individual components, while in Matutes and Regibeau (1992), firms can price it differently.

⁹ Avenali et al. (2013) focus on technological bundling in which bundling two products improves the quality of a system.

I use a model of “competitive bottlenecks” (Armstrong 2006). The mass of consumers is normalized to 1, uniformly distributed along the unit interval; they single-home. Let x denote a consumer’s location on the unit interval between A_1 and A_2 , and let t denote the linear transportation cost in the distance needed to travel to each platform. Consumers see an identical intrinsic value, v , in the two platforms, which is large enough so that there is full market coverage.¹⁰ Consumers have a taste for application variety: The availability of each additional developer positively generates additional utility α for each consumer, where $\alpha \geq 0$. A consumer located at x is indifferent towards the platforms if

$$v + \alpha n_{A_1}^D - p_{A_1}^C - tx = v + \alpha n_{A_2}^D - p_{A_2}^C - t(1 - x). \quad (1)$$

Consumers have unit demand in market B . Let θ denote the consumer’s marginal utility of q_{Bi} . Assume that $\theta = 1$ for all consumers—i.e. they are homogeneous with respect to the valuation of product Bi .

From the perspective of developers, the two platforms are identical. The mass of developers is normalized to 1. They differ in the cost of listing applications, denoted by y , and are uniformly distributed along the segment $[0, 1]$. All applications are independent of one another, and each developer can list at most one application on each platform. Developers can multi-home,¹¹ but the fixed listing cost per platform is incurred twice if a developer joins both platforms. Each developer gains an additional utility of β from each consumer available on A_i , where $\beta > 0$. The utility of developer y for joining platform A_i is $u_{A_i}^D(y) = \beta n_{A_i}^C - p_{A_i}^D - y$.

Assume that the following conditions hold throughout this paper:

Assumption A1. $t > \underline{t} = \frac{\alpha^2}{6} + \frac{2\alpha\beta}{3} + \frac{\beta^2}{6}$.

Under Assumption A1, the conditions for a unique and stable equilibrium ($t > \frac{\beta^2}{6} + \frac{\alpha^2}{6} + \frac{2\alpha\beta}{3}$) and second order condition ($t > \alpha\beta$) are satisfied.

Assumption A2. $\alpha + \beta < 2$.

Assumption A2 rules out the corner solution that the developer demand for each platform is 1.

Assumption A3. $\beta > \alpha$.

¹⁰ The intrinsic value cannot be realized without interacting with developers through a platform.

¹¹ Survey data shows that, on average, mobile developers use 2.6 mobile platforms (VisionMobile 2013).

This paper only considers the case in which developers value consumer participation more.¹²

The timing of the proposed two-stage game is as follows. In stage 1, firms decide whether to sell the two products as a bundle. The decisions are publicly observable. In stage 2, firms simultaneously set prices; consumers and developers make participation decisions. The equilibrium concept is subgame-perfect Nash equilibrium; I solve this game by backward induction.

4 Platform Competition

4.1 Price Competition Stage

At stage 2, firms compete in prices given the bundling decision made in stage 1. I thus analyze the following subgames.

4.1.1 Separate Selling

First consider the subgame in which firms decide not to bundle in stage 1. Consumers make consumption decisions in the two markets independently.

Lemma 1. *If both firms decide not to bundle in stage 1, the prices and demands of the unique symmetric equilibrium in market A are as follows:*

$$\begin{aligned} p_{Ai}^C &= t - \frac{\beta^2}{4} - \frac{3\alpha\beta}{4}, & n_{Ai}^C &= \frac{1}{2}, \\ p_{Ai}^D &= \frac{\beta}{4} - \frac{\alpha}{4}, & n_{Ai}^D &= \frac{\beta}{4} + \frac{\alpha}{4}. \end{aligned}$$

In market B, the equilibrium prices are given by $p_{B1}^ = \delta$ and $p_{B2}^* = 0$. All consumers consume B1. Each firm's profit is given by*

$$\Pi_1^* = \frac{t}{2} - \frac{\alpha^2}{16} - \frac{3\alpha\beta}{8} - \frac{\beta^2}{16} + \delta, \quad \Pi_2^* = \frac{t}{2} - \frac{\alpha^2}{16} - \frac{3\alpha\beta}{8} - \frac{\beta^2}{16}.$$

“Mix and match” allows all consumers to assemble a system that is close to their ideal. In market A, both platforms split the consumer market evenly. The equilibrium price is the standard Hotelling price adjusted downward by $\frac{\beta^2}{4} + \frac{3\alpha\beta}{4}$.

¹² Assumption A3 is consistent with the findings in previous literature that the single-homing side of competitive bottlenecks are treated favorably (Rochet and Tirole 2003; Armstrong 2006) and the reality that the developer side is the profit-making side.

The adjustment term measures the benefits of attracting an additional consumer to the platform and can be further broken down into two factors, $\beta(\frac{\beta}{4} + \frac{3\alpha}{4})$. The first factor means that the platform attracts β additional developers when it has an additional consumer. The second factor is the profit that the platform earns from each additional developer. The additional developer pays $\frac{\beta}{4} - \frac{\alpha}{4}$ for access to the platform but also attracts α consumers. Platforms subsidize consumers for participation when consumer preferences for platforms are weak ($t < \frac{\beta^2}{4} + \frac{3\alpha\beta}{4}$).

There is no full market coverage on the developer side. Developers with low listing costs multi-home (i.e. $y \leq \frac{\beta}{4} + \frac{\alpha}{4}$); the ones with higher costs do not participate (i.e. $y > \frac{\beta}{4} + \frac{\alpha}{4}$). The equilibrium developer price is the monopoly pricing $\frac{\beta}{4}$ adjusted downward by $\frac{\alpha}{4}$, which is the added benefit that an extra developer brings to the platform from attracting consumers.

4.1.2 Unilateral Bundling by Firm 1

Consider the subgame of firm 1's unilateral bundling. A consumer may either consume the $(A1, B1)$ bundle or assemble a system with $A2$ and $B2$. I denote the variables under the bundling scheme with a tilde ($\tilde{\cdot}$). Let \tilde{P}_1 and \tilde{n}_1 denote the price and consumer demand of the $(A1, B1)$ bundle, respectively. Firm 1 has more incentive to lower \tilde{P}_1 because doing so stimulates the demand for both components of the bundle. A consumer located at x is indifferent toward the $(A1, B1)$ bundle and assembling a system with $A2$ and $B2$ if

$$v + \alpha \tilde{n}_{A1}^D - \tilde{P}_1 - tx + z = v + \alpha \tilde{n}_{A2}^D - \tilde{p}_{A2}^C - t(1-x) + z - \delta - \tilde{p}_{B2}. \quad (2)$$

For expositional simplicity, let $\Phi = 6t - \beta^2 - 4\alpha\beta - \alpha^2$ for the remainder of the paper.¹³

Lemma 2. *If firm 1 decides to bundle while firm 2 decides not to bundle in stage 1, the equilibrium prices are given by:*

$$\begin{aligned} \tilde{p}_1^* &= t - \frac{\beta^2}{4} - \frac{3\alpha\beta}{4} + \frac{\delta(4t - 3\alpha\beta - \beta^2)}{2\Phi}, \quad \tilde{p}_{A2}^* = t - \frac{\beta^2}{4} - \frac{3\alpha\beta}{4} - \frac{\delta(4t - \beta^2 - 3\alpha\beta)}{2\Phi}, \\ \tilde{p}_{A1}^{D*} &= \left(\frac{\beta - \alpha}{2}\right) \left(\frac{1}{2} + \frac{\delta}{\Phi}\right), \quad \tilde{p}_{A2}^{D*} = \left(\frac{\beta - \alpha}{2}\right) \left(\frac{1}{2} - \frac{\delta}{\Phi}\right), \quad \tilde{p}_{B2}^* = 0. \end{aligned}$$

¹³ Note that $\Phi > 0$ because $t > t$.

Each system's demands are given by:

$$\begin{aligned}\tilde{n}_1^* &= \frac{1}{2} + \frac{\delta}{\Phi}, & \tilde{n}_{A2}^{C^*} &= \tilde{n}_{B2}^* = \frac{1}{2} - \frac{\delta}{\Phi}, \\ \tilde{n}_{A1}^{D^*} &= \left(\frac{\beta + \alpha}{2}\right) \left(\frac{1}{2} + \frac{\delta}{\Phi}\right), & \tilde{n}_{A2}^{D^*} &= \left(\frac{\beta + \alpha}{2}\right) \left(\frac{1}{2} - \frac{\delta}{\Phi}\right).\end{aligned}$$

Each firm's equilibrium profit is given by:

$$\tilde{\Pi}_1^* = \frac{8t - \beta^2 - \alpha^2 - 6\alpha\beta(\Phi + 2\delta)^2}{16\Phi^2}, \quad \tilde{\Pi}_2^* = \frac{8t - \beta^2 - \alpha^2 - 6\alpha\beta(\Phi - 2\delta)^2}{16\Phi^2}.$$

Bundling allows firm 1 to commit to a lower consumer price for its system and attracts consumers who prefer platform A1 and slightly prefer platform A2. The higher vertical differentiation there is in market B, the lower bundle price firm 1 commits to, and the more consumer demand bundling stimulates. Consequently, bundling equips firm 1 with more market power on the developer side, which leads to a higher price and higher participation on the developer side. The effect of a marginal increase in δ on firm 1's profit can be broken down into both direct and strategic effects, as follows:

$$\frac{d\tilde{\Pi}_1}{d\delta} = \frac{\partial\tilde{\Pi}_1}{\partial\delta} + \frac{\partial\tilde{\Pi}_1}{\partial\tilde{p}_{A2}^{C^*}} \frac{d\tilde{p}_{A2}^{C^*}}{d\delta} + \frac{\partial\tilde{\Pi}_1}{\partial\tilde{p}_{A2}^{D^*}} \frac{d\tilde{p}_{A2}^{D^*}}{d\delta}.$$

Note that the direct effect of a marginal increase in δ on $\tilde{\Pi}_1$ is $\frac{\partial\tilde{\Pi}_1}{\partial\delta} = \tilde{n}_1$. It indicates that B1 is sold only as a component of the bundle. The strategic effects work through the two-sided prices of the rival platform. They show how the rival platform reacts to bundling on two-sided pricing and whether these reactions intensify or soften price competition. The strategic effect on the consumer side is:

$$\frac{\partial\tilde{\Pi}_1}{\partial\tilde{p}_{A2}^{C^*}} \frac{d\tilde{p}_{A2}^{C^*}}{d\delta} = (\tilde{P}_1 + \beta\tilde{p}_{A1}^D) \frac{\partial\tilde{n}_1}{\partial\tilde{p}_{A2}^{C^*}} \frac{-(4t - \beta^2 - 3\alpha\beta)}{2\Phi}.$$

Whether firm 1's bundling intensifies or softens price competition on the consumer side depends on the relationship between consumer platform preferences and the benefits of attracting an additional consumer. When consumer preferences for platforms are weak ($t < t < \frac{\beta^2}{4} + \frac{3\alpha\beta}{4}$), platforms implement cross subsidies from developers to consumers. Consumer prices are strategic substitutes ($\frac{\partial\tilde{P}_1(\tilde{p}_2^C)}{\partial\tilde{p}_2^C} < 0$): Platform A2 moves against the rival's pricing movement—i.e. reducing consumer subsidies in response to bundling. Price competition is thus softened on the consumer side ($\frac{\partial\tilde{\Pi}_1}{\partial\tilde{p}_2^C} \frac{d\tilde{p}_2^C}{d\delta} > 0$). The positive feedback effect amplifies

bundling’s effectiveness in stimulating consumer demand: The more application variety, the more effectively bundling stimulates consumer demand, insofar as it reduces the magnitude of the discount on the bundle price for bundling to be profitable for firm 1. Thus, platform A2 is better off reducing consumer subsidies, as it is very costly for it to compete with the aggressive rival. When consumer preferences for platforms get more intense ($t > \frac{\beta^2}{4} + \frac{3\alpha\beta}{4}$), platforms charge positive consumer prices. Consumer prices are strategic complements ($\frac{\partial \tilde{P}_1(\tilde{p}_2^c)}{\partial \tilde{p}_2^c} > 0$): Platform A2 follows the rival’s pricing movement to increase consumer subsidies in response to bundling. Price competition is thus intensified on the consumer side ($\frac{\partial \tilde{\Pi}_1}{\partial \tilde{p}_2^c} \frac{d\tilde{p}_2^c}{d\delta} < 0$).

The strategic effect on the developer side is:

$$\frac{\partial \tilde{\Pi}_1}{\partial \tilde{p}_{A2}^D} \frac{d\tilde{p}_{A2}^D}{d\delta} = -(\tilde{P}_1 + \beta \tilde{p}_{A1}^D) \frac{\partial \tilde{n}_1}{\partial \tilde{p}_{A2}^D} \frac{\beta - \alpha}{2\Phi} < 0.$$

In response to firm 1’s bundling practice, platform A2 lowers its developer price, as it becomes less valuable to developers; price competition is thus intensified on the developer side. In Figure 1, the dotted lines depict the speed of Π_1 increasing in δ under separate selling, which is 1; the solid curves depict the speed of Π_1 increasing in δ under bundling. When $\underline{t} < t < \frac{\beta^2}{4} + \frac{3\alpha\beta}{4}$ (Figure 1A), Π_1 increases at a speed faster than \tilde{n}_1 : the speed is either faster than 1 or slower than 1 but faster than \tilde{n}_1 . In other words, either bundling is profitable for any value of δ , or it is profitable only when δ is of a significant value. When $t > \frac{\beta^2}{4} + \frac{3\alpha\beta}{4}$ (Figure 1B), Π_1 increases in δ at a speed that is strictly slower than \tilde{n}_1 . As a result, bundling cannot be profitable for firm 1.

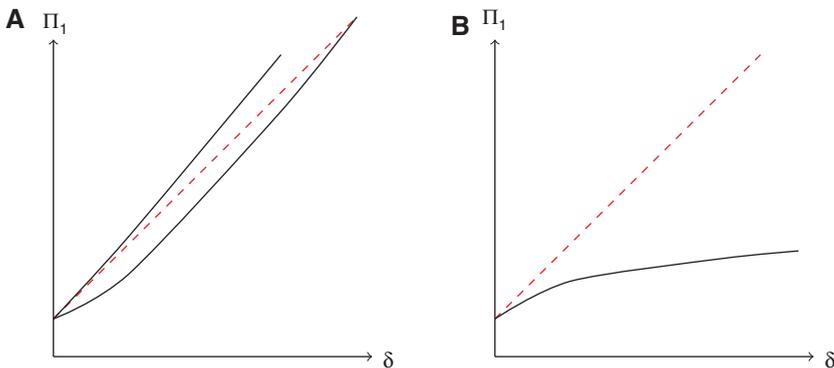


Figure 1: The Impact of Bundling on Firm 1’s Profit. (A) Strategic Substitutes. (B) Strategic Complements.

Bundling only shows strategic effects on firm 2's profit. Firm 1's bundling practice intensifies price competition on the consumer side for platform A2—i.e. $\frac{\partial \tilde{\pi}_2}{\partial \tilde{p}_1} \frac{d\tilde{p}_1^*}{d\delta} > 0$. Bundling also makes platform A1 more valuable to developers, so its developer price increases. Thus, price competition is softened on the developer side for platform A2—i.e. $\frac{\partial \tilde{\pi}_2}{\partial \tilde{p}_{A1}^D} \frac{d\tilde{p}_{A1}^*}{d\delta} > 0$. The overall effect of a marginal increase in δ on firm 2's profit is negative. Hence, bundling is detrimental to firm 2's profit.

As δ increases, firm 1 behaves more aggressively, and platform A2 further reduces subsidies. Eventually, platform A2's market share decreases to zero, and bundling leads to the foreclosure of firm 2. Sufficient vertical differentiation in market *B* allows firm 1 to commit to a large discount on the bundle price, so that even the consumer that has the most intense preference for platform A2 would choose the bundle.

4.1.3 Retaliatory Bundling

Now consider the subgame of unilateral bundling by firm 2. In stage 1, firm 2 decides to bundle, while firm 1 decides to sell the components separately. Let \tilde{P}_2 and \tilde{n}_2 denote the price and consumer demand of the (A2, B2) bundle, respectively. A consumer located at x is indifferent toward assembling a system that consists of firm 1's two components and the (A2, B2) bundle if

$$v + \alpha \tilde{n}_{A1}^D - \tilde{p}_{A1}^C - tx + z - \tilde{p}_{B1} = v + \alpha \tilde{n}_{A2}^D - \tilde{P}_2 - t(1-x) + z - \delta,$$

which is equivalent to Eq. 1. Consumers' utility from market *B* has no effect on the demand of platforms even if firm 2 introduces the bundle. Thus, given that firm 1 decides not to bundle in stage 1, firm 2 cannot commit to a more aggressive pricing strategy by bundling the platform with the inferior complementary product.

Next, consider the subgame of bilateral bundling. Suppose that both firms decide to bundle in stage 1. A consumer located at x is indifferent towards the two bundles if

$$v + \alpha \tilde{n}_{A1}^D - \tilde{P}_1 - tx + z = v + \alpha \tilde{n}_{A2}^D - \tilde{P}_2 - t(1-x) + z - \delta,$$

which is equivalent to Eq. 2. Firm 2 cannot commit to a more aggressive pricing strategy for consumers by bundling the platform with the inferior complementary good regardless of firm 1's bundling decision.

4.2 Bundling Strategy

Let $t_1 = \frac{5\alpha^2}{24} + \frac{7\alpha\beta}{12} + \frac{5\beta^2}{24}$, $t_2 = \frac{3\alpha^2}{16} + \frac{5\alpha\beta}{8} + \frac{3\beta^2}{16}$ and $\delta_1 = \frac{\Phi(16t - 3\alpha^2 - 10\alpha\beta - 3\beta^2)}{8t - \alpha^2 - 6\alpha\beta - \beta^2}$.¹⁴
The firms' bundling strategies in stage 1 are as follows.

Proposition 1. *Consider the model in which consumers are homogeneous with respect to the valuation of B_i . Regardless of firm 2's bundling decision, firm 1's bundling strategy is characterized as follows:*

For $\delta < \delta^* = \frac{\Phi}{2}$, firm 1's bundling practice does not lead to the foreclosure of firm 2.

- (i) *When consumer preferences for platforms are very weak—i.e. $\underline{t} < t \leq t_2$, bundling is profitable for firm 1 regardless of the value of δ .*
- (ii) *When consumer preferences for platforms get more intense—i.e. $t_2 < t \leq t_1$, bundling is profitable for firm 1 when $\delta \geq \delta_1$.*
- (iii) *Bundling reduces firm 1's profit otherwise.*

Bundling is detrimental to firm 2's profit.

For $\delta \geq \delta^*$, firm 1's bundling practice leads to the foreclosure of firm 2 and is profitable for firm 1 if $\underline{t} < t \leq t_1$.

Firm 2's bundling decision has no effect on competition.

The gray area in Figure 2 depicts the set of parameters within which bundling emerges as a profitable strategy for firm 1 in equilibrium. To generate sufficient additional profit on the developer side, bundling needs to steal sufficient consumer demand from the rival platform and expand developer demand sufficiently. When consumer preferences for platforms are weak, bundling stimulates sufficient consumer demand even with a small discount on \tilde{P}_1 , so bundling is profitable for firm 1 regardless of the value of δ . When consumer preferences for platforms get more intense, firm 1 has to offer a larger discount on \tilde{P}_1 in order to stimulate sufficient consumer demand; in other words, the level of vertical differentiation needs to be high enough ($\delta \geq \delta_1$). When consumer preferences for platforms get very intense, it is impossible for firm 1 to stimulate sufficient consumer demand for bundling to be profitable. When bundling leads to the foreclosure of firm 2, it can only be profitable when the platforms are not very differentiated, so that firm 1 does not need to compensate consumers too heavily for their distaste for platform A1.

I conclude that firm 1 can leverage its quality advantage from the complementary good market to the two-sided platform market through bundling when it

¹⁴ Note that $t_2 < t_1$ holds.

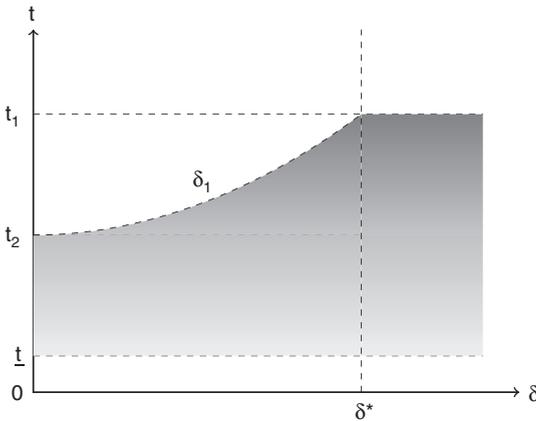


Figure 2: Firm 1's Bundling Strategy.

competes with a horizontally differentiated platform. Bundling gives firm 1 a competitive advantage in the platform market that cannot be eroded by the bundling practice of firm 2. The results provide a theory that can explain the observations from the mobile application store market. It contributes to better understanding of Google's anti-competitive bundling behavior. It helps to explain why the alternative app stores that run on Android have such a small market share. It also explains why a firm (e.g. Samsung) that owns an inferior complementary good may not want to build a proprietary ecosystem by bundling the platform with the inferior complementary good.

Note that if firm 1 practices tying, it also sells $B1$ on a stand-alone basis, so consumers can choose a system that is close to their ideal. Thus, there is no difference between tying and untying in terms of what is offered. This result contradicts Amelio and Jullien (2012) in the absence of the non-negative price constraint.

4.3 Welfare Analysis

Bundling affects consumer welfare through the prices of the two systems, the application variety on offer, and the distance between a consumer's ideal product in each market and the one she or he consumes.

Proposition 2. *Firm 1's bundling practice improves consumer welfare.*

Bundling may improve consumer welfare when the indirect network effects are at play. The consumers who purchase the $(A1, B1)$ bundle experience positive effects via a lower bundle price and more application variety. However, for the

consumers that consume a less-preferred platform, such a mismatch creates a loss. Bundling negatively impacts the consumers who consume the alternative system: They receive a lower subsidy for platform A2 and have access to fewer applications. The positive effects dominate. Indeed, the higher vertical differentiation in market B , the higher consumer surplus under bundling. When bundling leads to the foreclosure of firm 2, bundling may still be welfare enhancing for consumers: All consumers enjoy a lower bundle price and more application variety, even though half of the consumers consume the less-preferred platform.

Let $\delta_2 = \frac{2\Phi^2}{20t-10\alpha\beta-\beta^2-\alpha^2}$ and $t_3 = \frac{3(\alpha+\beta)^2}{4}$. The following proposition characterizes the impact of firm 1's bundling practice on social welfare.

Proposition 3. *Firm 1's bundling practice is socially beneficial if $\underline{t} < t \leq t_3$ and $\delta \geq \delta_2$. When bundling leads to the foreclosure of firm 2 (i.e. $\delta \geq \delta^* = \frac{\Phi}{2}$), bundling is socially beneficial if $\underline{t} < t \leq t_3$. Otherwise, bundling reduces social welfare.*

There are two ways in which firm 1's bundling practice positively affects social welfare. First, it enhances participation on both sides of the bundling platform, resulting in a higher utility from the indirect network effects. Second, it reduces the mass of developers who multi-home, and hence the cost of multi-homing. However, bundling also leads to three sources of inefficiency. First, two-sided users of platform A2 suffer a utility loss because of lower participation on both sides. Second, some consumers end up consuming the platform that is further from their ideal. Third, the alternative system consists of the inferior complementary good B2. Proposition 3 shows that for bundling to be socially beneficial, there has to be sufficient vertical differentiation in market B , and the platforms cannot be too differentiated. High vertical differentiation in market B allows firm 1 to stimulate sufficient consumer demand. Consequently, it leads to a significant welfare gain from the indirect network effects and a significant reduction in the developers' cost of multi-homing. High consumer demand for the (A1, B1) bundle also indicates a small welfare loss due to low consumer demand for B2. Note that when bundling leads to the foreclosure of firm 2, the indirect network effects can be fully internalized, as all users in market A interact on platform A1, and thus there is no cost of multi-homing incurred on the developer side. The inefficiency of bundling stems from the fact that half of the consumers consume the less-preferred platform.

To see the desirability of bundling, let $t_4 = \frac{3\alpha^2}{28} + \frac{\alpha\beta}{2} + \frac{3\beta^2}{28} + \frac{\sqrt{22\alpha^4+56\alpha^3\beta+100\alpha^2\beta^2+56\alpha\beta^3+22\beta^4}}{56}$.¹⁵ For $\underline{t} < t \leq t_4$ and $\delta \geq \delta_2$, $t_4 < t \leq t_1$ and

¹⁵ Note that $t_4 < t_1 < t_3$ holds.

$\delta \geq \delta_1$, bundling is socially desirable when it emerges as the equilibrium strategy for firm 1. Note that when platforms are not too differentiated, consumers are more responsive to a discount on the bundle price, and it is relatively easy for bundling to be profitable. However, for bundling to be socially beneficial requires certain level of incremental indirect network effects and decreased multi-homing costs on the developer side, and hence a certain level of δ . For $t_4 < t \leq t_1$ and $\delta_2 \leq \delta < \delta_1$, $t_1 < t \leq t_3$ and $\delta \geq \delta_2$, bundling is socially desirable but not the equilibrium strategy for firm 1. When platform differentiation is high, it is more difficult for bundling to be profitable for firm 1 though it may be socially beneficial. This is because firm 1 needs to offer a larger discount on the bundle price to compensate consumers for their preferences for platforms.

It is important to note that the benefits of bundling largely stem from the utility gain of the indirect network effects, as bundling raises participation on both sides of the bundling platform. Bundling generates a tipping toward the bundling platform, and the positive feedback loop between the two sides makes the rival platform even more vulnerable. Consequently, bundling can limit consumers' choices. Thus, I suggest that competition authorities scrutinize bundling practices in detail when two-sided markets are involved, so they may strike a balance between coordinating two-sided demands and increasing the level of choice available to consumers.

Even though tying would not emerge as an equilibrium strategy for firm 1, competition authorities can require the mandated stand-alone sale of $B1$ in addition to the bundle.¹⁶ However, firm 1 can effectively practice pure bundling if it raises the stand-alone price for $B1$; in that case, the consumers who do not consume the $(A1, B1)$ bundle would consume the inferior complementary good $B2$. Thus, I argue that the mandated stand-alone sale should come along with a price cap. The price cap requires that firm 1 not raise the price of $B1$ under tying above the equilibrium price under separate selling.¹⁷ Such requirements improve social welfare.

Provided that the developer side is the profit-making side in the platform market, I analyze and discuss how the degree of two-sidedness of the platform

16 Competition authorities can potentially require mandated mixed bundling – i.e. the firm offers the bundle and stand-alone sales of both components. However, in the setting of this paper, the dominant strategy for the consumers who prefer platform $A1$ is to consume the $(A1, B1)$ bundle. Thus, mixed bundling is reduced to tying.

17 Avenali et al. (2013) proposes a “monopoly component price” test that controls the stand-alone price of the monopoly component under mixed bundling. They show that it preserves efficiencies from both bundling and quality investment.

market—i.e. β , affects firm 1's incentive to bundle, consumer welfare and social welfare, taking $\alpha = 0$.

Lemma 3. *When the interior equilibrium prevails under bundling—i.e. $\delta < \delta^* = \frac{\Phi}{2}$, firm 1's profit in the bundling equilibrium is non-monotonic in β . Consumer surplus and total surplus increase in β .*

To determine how β affects firm 1's profit under bundling, it is necessary to consider two effects. The first effect is negative on the profit from the consumer side: keeping other variables constant, the equilibrium bundle price decreases as β increases, and the demand for bundle increases in β . This happens because consumers are more valuable to firm 1 as developers highly value consumer participation. Firm 1 behaves more aggressively to stimulate consumer demand. The second effect is positive: the profit from the developer side increases as β increases. As a result, firm 1's profit under bundling is non-monotonic in β . Indeed, firm 1's profit decreases in β when $\delta < \frac{(6t-\beta)^2}{20t-2\beta^2}$ and increases in β when $\delta > \frac{(6t-\beta)^2}{20t-2\beta^2}$. Keeping other variables constant and taking $\alpha = 0$, consumer surplus is determined by the bundle price and the match between consumer preferences and platforms. When β rises, firm 1 has more incentive to lower the bundle price to stimulate consumer demand. At the same time, a higher degree of two-sidedness leads to more consumers consuming the less-preferred platform. The positive effect dominates, so consumer surplus is positively correlated with β . Keeping other variables constant, a higher degree of two-sidedness also leads to higher developer surplus. The positive effect of a higher degree of two-sidedness dominates. Total surplus is positively correlated with β as well.

5 Robustness Check

This section considers the robustness of the insights obtained from the main model.

5.1 Extension to Independent Goods

Now I study firm 1's decision to bundle a two-sided platform with an independent good. The two markets are independent for consumers in the sense that each consumer's utility in one market is independent of her or his utility in the other market. This modification can show that the results obtained in the main model are robust when the two components of the bundle are independent goods.

I modify the model with independent products in Whinston (1990) by incorporating two-sidedness in the differentiated market. Assume that market A remains as described in the main model. Firm 1 is a monopoly in market B . Consumers have a homogeneous willingness to pay for product B , denoted by z . Absent bundling, the equilibrium of market A is the same as that of the main model. In market B , all consumers purchase product B at price $p_B = z$.¹⁸ If firm 1 decides to bundle, a consumer consumes either the $(A1, B)$ bundle or platform $A2$. Let \bar{P}_1 denote the price for the $(A1, B)$ bundle. The implicit price of access to platform $A1$ is $\bar{p}_{A1}^C = \bar{P}_1 - z$. A consumer located at x is indifferent toward the $(A1, B)$ bundle and platform $A2$ if

$$v + \alpha \bar{n}_{A1}^D - \bar{P}_1 - tx + z = v + \alpha \bar{n}_{A2}^D - \bar{p}_{A2}^C - t(1 - x),$$

which is very similar to Eq. 2. Thus, I confirm that firm 1 can commit to a more aggressive pricing strategy by bundling a two-sided platform with an independent good as it faces competition in the platform market. Bundling's effectiveness in stimulating demands in the two-sided market depends on consumers' willingness to pay for the independent good.

5.2 Economies of Scope

The analysis in this paper is carried out under the assumption that there are no economies of scope in multi-homing for developers. This assumption works in favor of profitable pure bundling. Some may argue that the cost of adapting an existing application to a second platform is lower than that of developing a new application from scratch. To see how the assumption of no economies of scope affects firm 1's bundling strategy, first consider the case in which the listing cost is incurred only once if a developer joins two active platforms. The developer effectively chooses between multi-homing and not participating. Consumers face the same set of applications on both platforms. The market-expansion effect induced by a lower consumer price through bundling is fully captured by both platforms. Firm 1 needs to commit to a substantially larger discount on the bundle price, because the bundling platform cannot offer more application variety. Now consider the case in which a portion of the listing cost is incurred again if a developer joins a second platform. If firm 1 commits to a lower bundle price through

18 The conclusion does not change if I assume that firm 1 has a superior independent good and firm 2 has an inferior one. Consumers are homogeneous with respect to the valuation of B_i .

bundling, some of the benefits of market expansion on the developer side are captured by the rival platform. Thus, bundling's effectiveness in stimulating two-sided demands is dampened when there are economies of scope in multi-homing for developers.

6 Conclusion

This paper examines the bundling strategy of two-product firms engaged in duopolistic competition in a two-sided platform market and a complementary good market. I model horizontal differentiation in the two-sided market and vertical differentiation in the complementary good market. Bundling works as a commitment device: It allows the firm that owns the superior complementary good to commit to a lower bundle price for consumers and consequently expands demand on the developer side in the platform market. Thus, the firm can leverage its quality advantage from the complementary good market to the platform market through bundling.

This paper carries an important message for policymakers. Bundling is usually considered anti-competitive and consequently detrimental to consumer welfare. However, when one of the markets involved is two sided, bundling may improve consumer welfare, largely because it promotes better internalization of the indirect network effects. Additionally, the positive feedback loop between the two sides amplifies the effectiveness of bundling in stimulating demands and makes the rival platform even more vulnerable, as bundling generates a tipping toward the bundling platform. Consequently, bundling can limit consumers' choices. The mandated stand-alone sale of the superior complementary good with a price cap can improve social welfare. Related to Google's anti-competitive behavior of bundling apps, I propose that giving competing apps equal access to Android pre-installation can improve social welfare. In light of the challenges of coordinating two-sided demands efficiently and increasing the level of choice available to consumers, I suggest that competition authorities scrutinize bundling practices involving two-sided markets in detail.

A Proofs of Propositions in Text

Proof of Proposition 1

Let $t_1 = \frac{5\alpha^2}{24} + \frac{7\alpha\beta}{12} + \frac{5\beta^2}{24}$, $t_2 = \frac{3\alpha^2}{16} + \frac{5\alpha\beta}{8} + \frac{3\beta^2}{16}$ and $\delta_1 = \frac{\Phi(16t - 3\alpha^2 - 10\alpha\beta - 3\beta^2)}{8t - \alpha^2 - 6\alpha\beta - \beta^2}$. First consider the scenario in which bundling does not lead to the foreclosure of firm

2—i.e. $\delta < \delta^* = \frac{\Phi}{2}$. Firm 1's profit change with bundling is given by:

$$\Delta\Pi_1 = -\frac{\delta}{4} \left[\frac{16t - 3\alpha^2 - 10\alpha\beta - 3\beta^2}{\Phi} - \frac{(8t - \alpha^2 - 6\alpha\beta - \beta^2)\delta}{\Phi^2} \right].$$

For bundling to be profitable for firm 1, the following condition must be satisfied:

$$\delta \geq \frac{(16t - 3\alpha^2 - 10\alpha\beta - 3\beta^2)\Phi}{8t - \alpha^2 - 6\alpha\beta - \beta^2} = \delta_1.$$

Note that when $\underline{t} < t \leq t_2$, $\delta_1 \leq 0$, so $\delta > \delta_1$ must be satisfied; when $t > t_2$, $\delta \geq \delta_1 > 0$. Additionally, $\delta < \frac{\Phi}{2}$ needs to be satisfied. When $\underline{t} < t \leq t_1$, $\delta_1 < \frac{\Phi}{2}$ holds. Hence, firm 1's bundling strategy in the interior equilibrium can be characterized as follows: When $\underline{t} < t \leq t_2$, bundling is profitable for firm 1 regardless of the value of δ ; when $t_2 < t \leq t_1$, bundling is profitable for firm 1 if $\delta \geq \delta_1$. Bundling leads to a change in firm 2's profit:

$$\Delta\Pi_2 = \frac{8t - \alpha^2 - 6\alpha\beta - \beta^2}{16} \left[\frac{(\Phi - 2\delta)^2}{\Phi^2} - 1 \right].$$

Note that $\text{sign}(\Delta\Pi_2) = \text{sign}\left[\frac{(\Phi - 2\delta)^2}{\Phi^2} - 1\right] = \text{sign}(\delta - \Phi)$. Hence, $\Delta\Pi_2 < 0$.

Then consider the scenario in which bundling leads to the foreclosure of firm 2—i.e. $\delta \geq \delta^* = \frac{\Phi}{2}$. Firm 1's profit change with bundling is given by:

$$\Delta\Pi_1 = -\frac{3t}{2} + \frac{5\alpha^2}{16} + \frac{7\alpha\beta}{8} + \frac{5\beta^2}{16}.$$

For bundling to be profitable for firm 1, the following condition must be satisfied:

$$t \leq \frac{5\alpha^2}{24} + \frac{7\alpha\beta}{12} + \frac{5\beta^2}{24} = t_1,$$

so that when $\underline{t} < t \leq t_1$, $\Delta\Pi_1 \geq 0$; when $t > t_1$, $\Delta\Pi_1 < 0$.

Proof of Proposition 2

Let W_C denote the consumer surplus. Under separate selling:

$$\begin{aligned} W_C &= \int_0^{n_{A1}^c} (v + \alpha n_{A1}^{D^*} - tx - p_{A1}^C) dx + \int_{1-n_{A2}^c}^1 [v + \alpha n_{A2}^{D^*} - t(1-x) - p_{A2}^C] dx \\ &\quad + z - \delta \\ &= v - \frac{5t}{4} + \frac{\alpha^2}{4} + \frac{\beta^2}{4} + \alpha\beta + z - \delta. \end{aligned}$$

First consider the scenario in which bundling does not lead to the foreclosure of firm 2—i.e. $\delta < \delta^* = \frac{\Phi}{2}$. In equilibrium, bundling leads to the change in consumer welfare:

$$\Delta W_C = \frac{\delta}{2} + \frac{t\delta^2}{\Phi^2} > 0.$$

Then consider the scenario in which bundling leads to the foreclosure of firm 2—i.e. $\delta \geq \delta^*$. In equilibrium, bundling leads to a change in consumer welfare:

$$\Delta W_C = \frac{7t}{4} - \frac{\alpha^2}{4} - \alpha\beta - \frac{\beta^2}{4}.$$

Note that $\Delta W_C > 0$ for all $t > \underline{t}$. ■

Proof of Proposition 3

Let W_T denote the total surplus. Under separate selling:

$$\begin{aligned} W_T &= W_C + \int_0^{n_{A1}^{D^*}} (\beta n_{A1}^{C^*} - p_{A1}^{D^*} - y) dy + \int_0^{n_{A2}^{D^*}} (\beta n_{A2}^{C^*} - p_{A2}^{D^*} - y) dy + \Pi_1^* + \Pi_2^* \\ &= v - \frac{t}{4} + \frac{3(\alpha + \beta)^2}{16} + \delta. \end{aligned}$$

First consider whether bundling is socially beneficial when bundling does not lead to the foreclosure of firm 2—i.e. $\delta < \delta^* = \frac{\Phi}{2}$. Let $\delta_2 = \frac{2\Phi^2}{20t - 10\alpha\beta - \beta^2 - \alpha^2}$ and $t_3 = \frac{3(\alpha + \beta)^2}{4}$. Bundling leads to a change in total surplus:

$$\Delta W_T = -\frac{\delta[2\Phi^2 - \delta(20t - 10\alpha\beta - \beta^2 - \alpha^2)]}{4\Phi^2}.$$

Note that $20t - 10\alpha\beta - \beta^2 - \alpha^2 > 0$ for all $t > \underline{t}$, and $\Delta W_T \geq 0$ when $\delta \geq \frac{2\Phi^2}{20t - 10\alpha\beta - \beta^2 - \alpha^2} = \delta_2$. Additionally, $\delta < \frac{\Phi}{2}$ needs to be satisfied. Hence, bundling is socially beneficial when $\underline{t} < t < t_3$ and $\delta \geq \delta_2$.

Then consider whether bundling is socially beneficial when bundling leads to the foreclosure of firm 2—i.e. $\delta \geq \delta^*$. In this scenario, bundling leads to a change in social welfare:

$$\Delta W_T = -\frac{t}{4} + \frac{3\alpha^2}{16} + \frac{3\alpha\beta}{8} + \frac{3\beta^2}{16}.$$

Therefore, when $\underline{t} < t \leq t_3$, bundling is socially desirable; when $t > t_3$, it is not. □

Proof of Lemma 3

When $\alpha = 0$, $\Phi = 6t - \beta^2$. When the interior equilibrium prevails under bundling, each firm's equilibrium profit is given by:

$$\tilde{\pi}_1^* = \frac{8t - \beta^2 - \alpha^2 - 6\alpha\beta(\Phi + 2\delta)^2}{16\Phi^2}, \quad \tilde{\pi}_2^* = \frac{8t - \beta^2 - \alpha^2 - 6\alpha\beta(\Phi - 2\delta)^2}{16\Phi^2}.$$

In equilibrium, consumer surplus and total surplus are given by

$$\tilde{W}_C^* = v - \frac{5t}{4} + \frac{\alpha^2}{4} + \frac{\beta^2}{4} + \alpha\beta + z - \frac{\delta}{2} + \frac{t\delta^2}{\Phi^2},$$

$$\tilde{W}_T^* = v - \frac{t}{4} + \frac{3(\alpha + \beta)^2}{16} + \delta - \frac{\delta[2\Phi^2 - \delta(20t - 10\alpha\beta - \beta^2 - \alpha^2)]}{4\Phi^2}.$$

The derivatives:

$$\frac{\partial \tilde{\pi}_1^*}{\partial \beta} = \frac{\beta[-\Phi^3 + 8t\delta\Phi + 4\delta^2(10t - \beta^2)]}{8\Phi^3}$$

$$\frac{\partial \tilde{\pi}_1^*}{\partial \beta} < 0 \text{ when } 0 < \delta < \frac{\Phi^2}{2(10t - \beta^2)}; \quad \frac{\partial \tilde{\pi}_1^*}{\partial \beta} > 0 \text{ when } \frac{\Phi^2}{2(10t - \beta^2)} < \delta < \frac{\Phi}{2}.$$

$$\frac{\partial \tilde{W}_C^*}{\partial \beta} = \frac{\beta(\Phi^3 + 8t\delta^2)}{2\Phi^3} > 0,$$

$$\frac{\partial \tilde{W}_T^*}{\partial \beta} = \frac{\beta[3\Phi^3 + 4\delta^2(34t - \beta^2)]}{8\Phi^3} > 0. \quad \square$$

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